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ORIGINAL ARTICLE

Heat and mass transfer in a Jeffrey fluid over a stretching sheet with heat source/sink

M. Qasim *

Department of Mathematics, COMSATS Institute of Information Technology, Park Road, Chak Shahzad, Islamabad, Pakistan

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Abstract This article studies the combined effect of heat and mass transfer in Jeffrey fluid over a stretching sheet in the presence of heat source/heat sink. The surface temperature and the concentration are assumed to vary according to power law form. The arising non-linear coupled partial differential equations are reduced to a set of coupled non-linear ordinary differential equations and then exact solutions are derived by power series method using Kummer's confluent hyper-geometric functions. The effects of emerging parameters on the velocity, temperature and concentration profiles are shown and examined. It is observed that the velocity increases with an increase in Deborah number. Further the temperature is a decreasing function of Deborah number. Thermal boundary layer thickness decreases by increasing the wall temperature and heat sink parameters.

1. Introduction

The non-linear fluid rheology has attracted the attention of recent researchers during the past few decades (see for example some studies [1–6] and several refs. therein). This interest stems because of extensive industrial and technological applications of flows of non-Newtonian fluids. It is well known that Newton's law of viscosity is inadequate for the description of flows in the non-Newtonian fluid dynamics. Moreover such fluids cannot be analyzed by a single constitutive relationship between shear stress and rate of strain. In view of diverse characteristics of non-Newtonian fluids in nature, there are several constitutive equations available in the literature. In general, the differential systems for such fluids are more complicated and much non-linear than for the viscous fluids.

Literature survey indicates that interest in the flows over a stretched surface has grown during the past few decades. These flows are arisen in metal and polymer extrusion, drawing of plastic sheets, cable coating, textiles and paper industries, etc. Sakiadis [7] performed the first study for the flow induced by a moving surface. The flow generated by a linear stretched sheet is examined by Crane [8]. Later, the stretching sheet flow has been studied by the several researchers for the sole effects of rotation, velocity and thermal slip conditions, heat and mass transfer, chemical reaction, MHD, suction/injection, different non-Newtonian fluids or possible combination of these

* Tel.: +92 51 9240823.
E-mail addresses: mq_qau@yahoo.com, mqasim@comsats.edu.pk.
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effects [9–17]. In the present attempt we explore the flow of a Jeffrey fluid [18–22] over a stretched sheet subject to power law temperature in the presence of heat source/heat sinks. In fact heat generation and absorption concepts in fluids have relevance in problems dealing with chemical reactions, geo-nuclear repositions and these concerned with dissociating fluids. Exact solutions have been established for flow temperature and concentration fields by power series methods using Kummer's confluent hyper-geometric functions [23]. Effects of pertinent parameters on the flow quantities of interest are seen and discussed.

2. Problem formulation

The constitutive equations for a Jeffrey fluid are given by [19]

$$\tau = -p\mathbf{I} + \mathbf{S},$$

$$\mathbf{S} = \frac{\mu}{1+\lambda}\left[\mathbf{R}_1 + \lambda_1\left(\frac{\partial \mathbf{R}_1}{\partial t} + \mathbf{V}\cdot\nabla\right)\mathbf{R}_1\right],$$

where τ is the Cauchy stress tensor, $\mathbf{S}$ is the extra stress tensor, μ is the dynamic viscosity, λ and λ_1 are the material parameters of Jeffrey fluid and $\mathbf{R}_1$ is the Rivlin–Ericksen tensor defined by

$$\mathbf{R}_1 = (\nabla V) + (\nabla V)'.$$

Here we discuss the steady two-dimensional flow of a Jeffrey fluid over a stretching sheet. Heat and mass transfer effects are considered. The sheet in XOZ plane is stretched in the x-direction such that the velocity component in x-direction varies linearly along it (Fig. 1). In the absence of viscous dissipation the governing boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\nu}{1+\lambda}\left[\frac{\partial^2 u}{\partial y^2} + \lambda_1\left(u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right)\right], \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty), \tag{3}$$

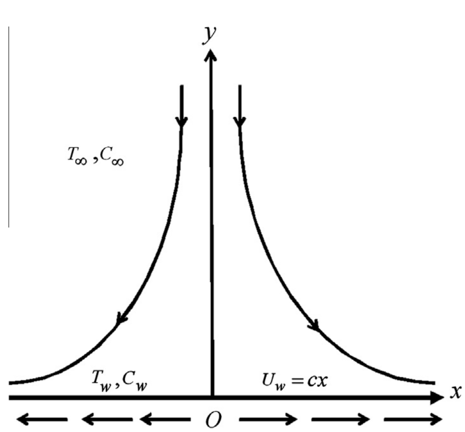

Figure 1 Physical model and coordinate system.

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}, \tag{4}$$

where u and v are the velocity components in the x- and y-directions respectively, T is the fluid temperature, C is concentration, ν is the kinematic viscosity, ρ is the fluid density, c_p is specific heat and D is diffusion coefficient. Here λ indicates the ratio of relaxation and retardation times and λ_1 is the relaxation time.

The boundary conditions for velocity components can be expressed as follows:

$$u = U_w(x) = cx, \quad v = 0, \quad y = 0,$$
$$u \to 0 \quad y \to \infty, \tag{5}$$

and the prescribed surface temperature and surface mass concentration can be put in the forms given below:

$$T = T_w = T_\infty + A_1\left(\frac{x}{l}\right)^m, \quad C = C_w = C_\infty + A_2\left(\frac{x}{l}\right)^m \quad \text{at} \quad y = 0,$$

$$T \to T_\infty, C \to C_\infty \quad \text{as} \quad y \to \infty, \tag{6}$$

in which A_1 and A_2 are the constants depending upon the properties of fluid, l is the characteristic length, T_w, C_w, T_∞, and C_∞ are temperature and species concentration at the wall and far away from the wall respectively. Setting

$$\eta = \sqrt{\frac{c}{\nu}}y, \quad u = cxf'(\eta), \quad v = -\sqrt{c\nu}f(\eta),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \tag{7}$$

Eq. (1) is satisfied and the non-dimensional form of Eqs. (2)–(6) is as follows:

$$f''' + (1+\lambda)(ff'' - f'^2) + \beta(f''^2 - ff'''') = 0, \tag{8}$$

$$\theta'' + \Pr(f\theta' - m\theta f' + \gamma\theta) = 0, \tag{9}$$

$$\phi'' + Sc(f\phi' - m\phi f') = 0, \tag{10}$$

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1,$$

$$f'(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0, \tag{11}$$

$$\beta = \lambda_1 c, \Pr = \frac{\rho c_p}{k}, Sc = \frac{\nu}{D}, \gamma = \frac{Q\nu}{\rho c_p}. \tag{12}$$

where prime denotes a differentiation with respect to η. Furthermore, β, Pr, Sc and γ are the Deborah, Prandtl, Schmidt and heat generation/absorption parameters respectively.

3. Exact analytical solutions

Let us seek a solution of Eq. (8) in the form

$$f(\eta) = \frac{1 - \exp(-\alpha\eta)}{\alpha}, \quad f'(\eta) = \exp(-\alpha\eta). \tag{13}$$

where

$$\alpha = \sqrt{\frac{1+\lambda}{1+\beta}} \tag{14}$$

Exact solution of Eq. (2) is obtained as

$$u = cx\exp(-\alpha\eta),\ v = -\sqrt{c\nu}\frac{1-\exp(-\alpha\eta)}{\alpha}. \tag{15}$$

Using Eq. (15) in Eqs. (9) and (10) we have

$$\theta'' + \Pr\left(\frac{1-\exp(-\alpha\eta)}{\alpha}\right)\theta' - m\Pr\theta\exp(-\alpha\eta) + \gamma\theta = 0, \tag{16}$$

$$\phi'' + Sc\left(\frac{1-\exp(-\alpha\eta)}{\alpha}\right)\phi' - mSc\phi\exp(-\alpha\eta) = 0, \tag{17}$$

To obtain the solution of Eqs. (16) and (17) respectively we introduce new variables ζ and ξ as follows

$$\zeta = \frac{\Pr}{\alpha^2}\exp(-\alpha\eta),\ \xi = \frac{Sc}{\alpha^2}\exp(-\alpha\eta), \tag{18}$$

Substituting (18) into Eqs. (16) and (17) one can write

$$\zeta\theta''(\zeta) + \left(1 - \frac{\Pr}{\alpha^2} + \zeta\right)\theta'(\zeta) - \left(m - \frac{\gamma}{\alpha^2\zeta}\right)\theta(\zeta) = 0, \tag{19}$$

$$\xi\phi''(\zeta) + \left(1 - \frac{Sc}{\alpha^2} + \xi\right)\phi'(\xi) - m\phi(\xi) = 0, \tag{20}$$

with the boundary conditions prescribed below

$$\theta\left(\zeta = \frac{\Pr}{\alpha^2}\right) = 1,\ \ \theta(\zeta = 0) = 0,$$
$$\phi\left(\xi = \frac{Sc}{\alpha^2}\right) = 1,\ \ \phi(\xi = 0) = 0. \tag{21}$$

The exact solutions of Eqs. (19) and (20) satisfying Eq. (21) are [16]

$$\theta(\zeta) = \left(\frac{\alpha^2}{\Pr}\zeta\right)^{\kappa_1+\kappa_2}\frac{{}^1F_1(\kappa_1+\kappa_2-m, 2\kappa_1+1, -\zeta)}{{}^1F_1(\kappa_1+\kappa_2-m, 2\kappa_1+1, -\frac{\Pr}{\alpha^2})},$$

$$\phi(\xi) = \left(\frac{\alpha^2}{Sc}\xi\right)^{\frac{Sc}{\alpha^2}}\frac{{}^1F_1\left(\frac{Sc}{\alpha^2}-m, 2\frac{Sc}{\alpha^2}+1, -\xi\right)}{{}^1F_1\left(\frac{Sc}{\alpha^2}-m, 2\frac{Sc}{\alpha^2}+1, -\frac{Sc}{\alpha^2}\right)}, \tag{22}$$

with $\kappa_1 = \frac{\Pr}{2\alpha^2}, \kappa_2 = \frac{\sqrt{\Pr^2-4\alpha^2\gamma}}{2\alpha^2}$ and ${}^1F_1(p,q,z)$ is the Kummer's function defined by

$${}^1F_1(a,b,z) = 1 + \sum_{n=1}^{\infty}\left(\frac{(a)_n}{(b)_n}\frac{z^n}{n!}\right). \tag{23}$$

Here $(a)_n$ and $(b)_n$ are Pochhammer's symbols defined by

$$(a)_n = a(a+1)(a+2)\ldots(a+n-1),$$
$$(b)_n = b(b+1)(b+2)\ldots(b+n-1), \tag{24}$$

In terms of variable η Eq. (22) can be written as

$$\theta(\eta) = \exp(-\alpha(\kappa_1+\kappa_2)\eta) \times \frac{{}^1F_1(\kappa_1+\kappa_2-m, 2\kappa_1+1, -\frac{\Pr}{\alpha^2}\exp(-\alpha\eta))}{{}^1F_1(\kappa_1+\kappa_2-m, 2\kappa_1+1, -\frac{\Pr}{\alpha^2})},$$

$$\phi(\eta) = \exp\left(-\frac{Sc}{\alpha}\eta\right)\frac{{}^1F_1\left(\frac{Sc}{\alpha^2}-m, 2\frac{Sc}{\alpha^2}+1, -\frac{Sc}{\alpha^2}\exp(-\alpha\eta)\right)}{{}^1F_1\left(\frac{Sc}{\alpha^2}-m, 2\frac{Sc}{\alpha^2}+1, -\frac{Sc}{\alpha^2}\right)}. \tag{25}$$

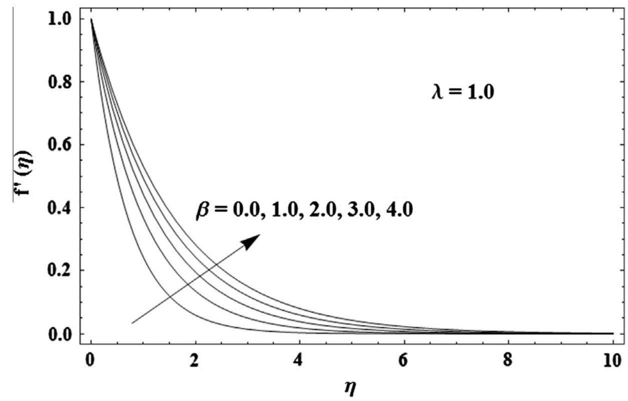

Figure 2 Influence of β on f'.

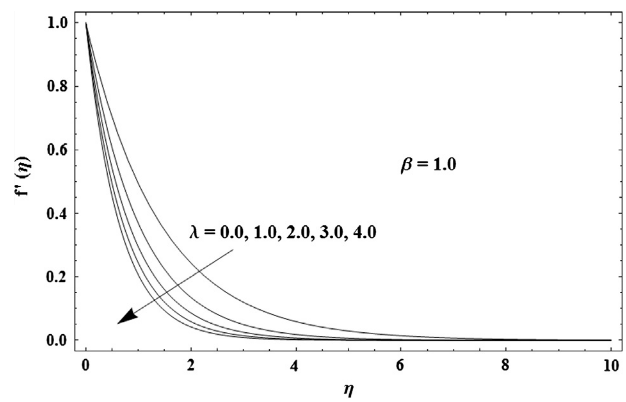

Figure 3 Influence of λ on f'.

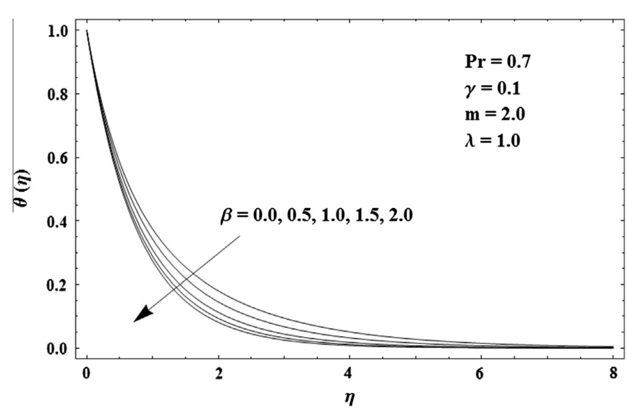

Figure 4 Influence of β on θ.

4. Results and discussion

In this section, the influence of emerging parameters on the velocity, temperature and concentration fields is studied. Figs. 2 and 3 describe the effects of β and λ on the velocity profile f'. From Fig. 2 it can be seen that the velocity field and boundary layer thickness are increasing functions of β. It is observed from Fig. 3 that the effect of λ is opposite to the effect of the Deborah number β. The effects of β, λ, Pr, m and γ on the temperature profile $\theta(\eta)$ are examined in Figs. 4–8. Fig. 4

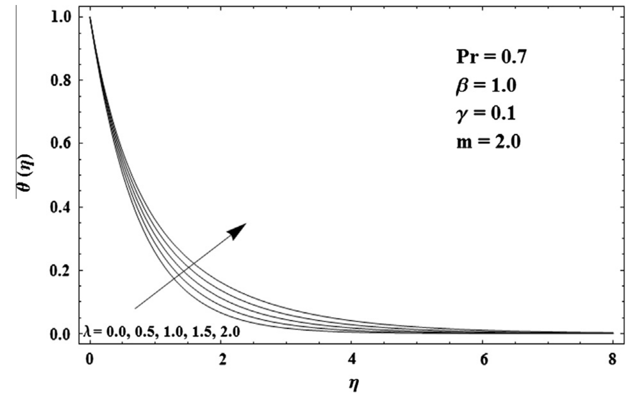

Figure 5 Influence of λ on θ.

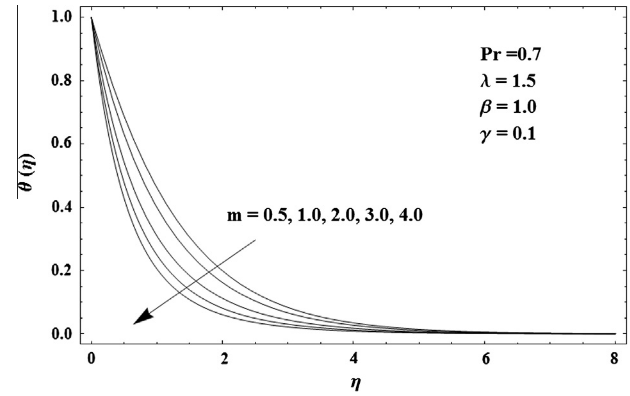

Figure 8 Influence of m on θ.

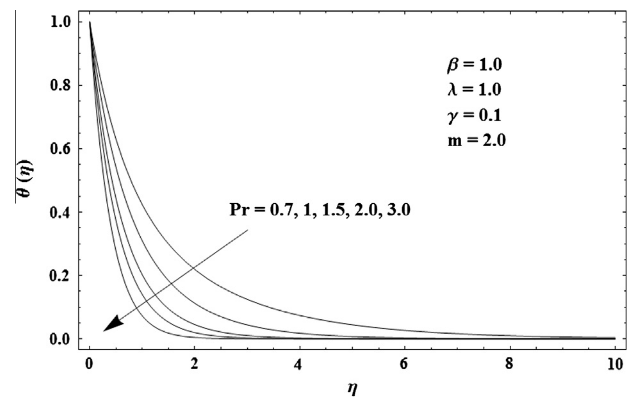

Figure 6 Influence of Pr on θ.

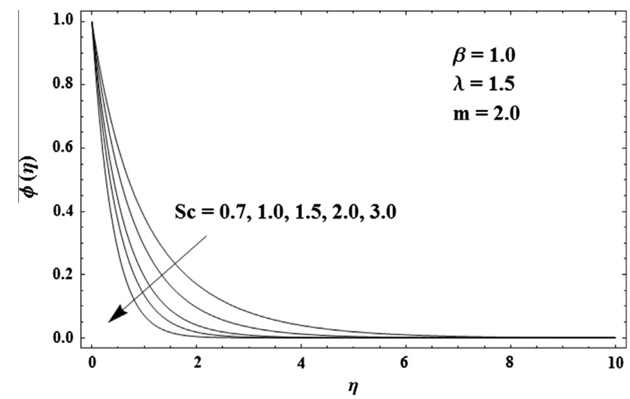

Figure 9 Influence of Sc on ϕ.

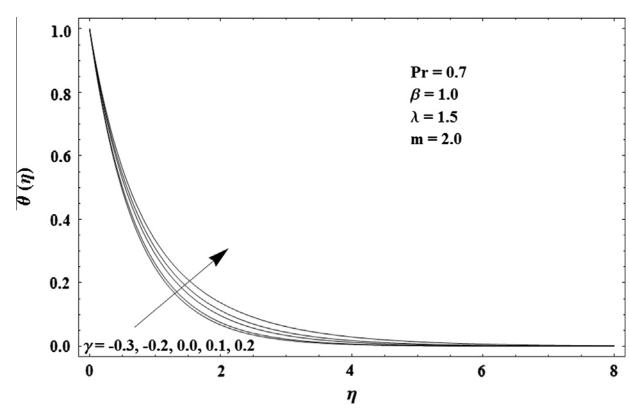

Figure 7 Influence of γ on θ.

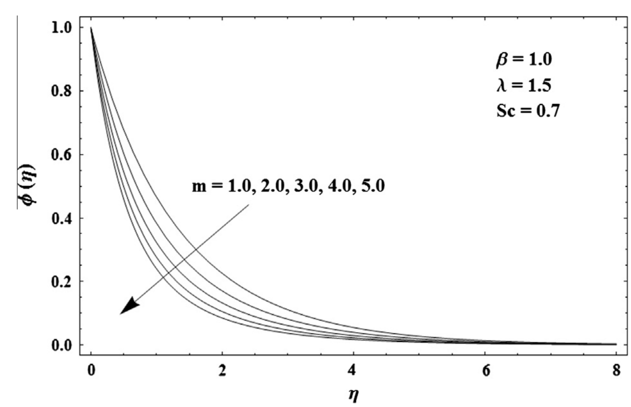

Figure 10 Influence of m on ϕ.

gives the variation in β on θ. Increasing the value of β decreases θ. The variation in λ on θ is given in Fig. 4. As λ increases, the temperature field increases. The variation in Prandtl number Pr on θ is shown in Fig. 6. The temperature field θ decreases when Pr increases. It is obvious that an increase in the values of Pr reduces the thermal diffusivity therefore thermal boundary layer thickness is decreasing function of Pr . The influence of heat generation absorption parameter on dimensionless temperature θ can be seen in Fig. 7. A gradual increase in heat source parameter increases the thermal boundary layer thickness which physically reveals the fact that an increase in the heat source/sink parameter means an increase in the heat generated inside the boundary layer which leads to higher temperature field. Fig. 8 plots the effect of surface temperature parameter m on the temperature profiles θ. It is observed that an increase in m decreases the thermal boundary layer thickness. The wall temperature parameter m plays a significant role in controlling the heat transfer. Fig. 9 depicts the graph of dimensionless concentration profiles for different values of Schmidt number Sc. We infer that concentration decreases with an increase in Schmidt number Sc. This means that hydrodynamic boundary layer is thicker than concentration boundary layer. This is due to the fact that an increase in the values of Schmidt number is to decrease the molecular diffusivity D that results in a decrease in thickness of concentration boundary layer. Thus for larger values of Schmidt number Sc the concentration of chemically reactive species is larger and lower for smaller values of Sc. As far as the effect of wall concentration parameter m on the concentration profile is concerned, it is obvious from Fig. 10 that an increase in m decreases the concentration.

5. Closing remarks

The present study describes the boundary layer flow of a Jeffrey fluid with heat and mass transfer effects. The main observations of this study are as follows:

- Increase in the value of Deborah number β leads to a decrease in the momentum boundary layer thickness and increase in the thermal boundary layer thickness.
- The effect of Deborah number β and parameter λ on the velocity is quite opposite.
- An increase in the value of Prandtl number Pr reduces the temperature and thermal boundary layer thickness.
- Both the thermal and concentration boundary layer thickness are decreasing function of m.
- An increase in the heat sink parameter γ results in lowering the temperature.

It is observed that concentration boundary layer thickness decreases by increasing Schmidt number Sc.

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